

($\xi\pi$)(EXAMPLE) 0.0.1. (SCRATCH: conjI sequent notation experiments)

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4  □ 1) Sequents with an assumes/shows theorem: Item a below is okay. Item
5      b is not because "[A;B]  $\Rightarrow$  (A  $\wedge$  B)" includes both the premise and
6      conclusion of the theorem, but when stated that way, it needs a
7      different proof.
8      a) A, B, <i(conjI),r(`A`),r(`B`)>  $\vdash$  "(A  $\wedge$  B)"
9      b)  $\vdash$  "[A;B]  $\Rightarrow$  (A  $\wedge$  B)", <i(conjI),r(`A`),r(`B`)> □
10 theorem conjI_1:
11     assumes "A" and "B"
12     shows "A  $\wedge$  B"
13                                     □ Goal: A  $\wedge$  B □
14 apply(intro conjI) □ Goal: A □
15 apply(rule `A`) □ Goal: B □
16   by(rule `B`)
17 thm conjI_1 □ [[A;?B]  $\Rightarrow$  ?A  $\wedge$  ?B □
18
19 □ 2) Sequents when assumptions are stated with meta implication: Item a
20     does not work because A, B needs to mean the same as 1 above. Item b
21     is wrong because [A;B] is not the premise. Items c and d are okay.
22     a) A, B, <i(conjI),a,a>  $\vdash$  "(A  $\wedge$  B)"
23     b) [A;B], <i(conjI),a,a>  $\vdash$  "(A  $\wedge$  B)"
24     c) <i(conjI),a,a>  $\vdash$  "[A;B]  $\Rightarrow$  (A  $\wedge$  B)"
25     d)  $\vdash$  "[A;B]  $\Rightarrow$  (A  $\wedge$  B)", <i(conjI),a,a> □
26 theorem conjI_2:
27     "[A;B]  $\Rightarrow$  (A  $\wedge$  B)"
28                                     □ Goal: [A;B]  $\Rightarrow$  A  $\wedge$  B □
29 apply(intro conjI) □ Goal: [A;B]  $\Rightarrow$  A □
30 apply(assumption) □ Goal: [A;B]  $\Rightarrow$  B □
31   by(assumption)
32 thm conjI_2 □ [[?A;?B]  $\Rightarrow$  ?A  $\wedge$  ?B □
33
34 □ 3)  $\vdash$  "[A;B]  $\Rightarrow$  (A  $\wedge$  B)", <e(conjI),a> □
35 theorem
36     "[A;B]  $\Rightarrow$  (A  $\wedge$  B)"
37                                     □ Goal: [A;B]  $\Rightarrow$  A  $\wedge$  B □
38 apply(elim conjI) □ Goal: B  $\Rightarrow$  B □
39   by(assumption)
40
41 □ 4) The backwards proof of conjI_1 shows how to do a forward proof. □
42 theorem
43     assumes "A" and "B"
44     shows "A  $\wedge$  B"
45 proof-
46   have "B"
47     by (rule `B`)
48   have "A"
49     by (rule `A`)
50   with `B` show
51     "A  $\wedge$  B"
52   by(intro conjI)
53 qed

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