$\left(\kappa\xi\right)$ (CounterX) 0.0.1. (cTest)

```
4
5    theory cTest
6    imports Complex_Main
7    begin
8
9    typedecl sT
10
11   consts inP :: "sT => sT => bool" (infixl "∈ι" 55)
12
13   abbreviation niP :: "sT => sT => bool" (infixl "∉ι"  55) where
14     "niP p q == ¬(p ∈ι q)"
15
16   axiomatization where
17     Ax_x^N: "(∀x. x ∈ι r ⟷ x ∈ι s) ⟷ r = s"
18
19   consts emS :: "sT" ("∅")
20
21   axiomatization where
22     Ax_emS: "(x ∉ι ∅)"
23
24   consts paS :: "sT => sT => sT"
25
26   syntax "_paS" :: "sT => sT => sT" ("({_,_})")
27     translations
28     "{r,s}" == "CONST paS r s"
29
30   axiomatization  where
31     Ax_paS: "x ∈ι {r,s} ⟷ (x = r ∨ x = s)"
32
33   theorem paS_is_unique:
34     "(∀x. x ∈ι r ⟷ (x = p ∨ x = q)) ⟷ r = {p,q}"
35     by(metis Ax_x^N Ax_paS)
36
37   definition siS :: "sT => sT" where
38     "siS r = paS r r"
39     notation
40     siS  ("({_})")
41
42   theorem siS_is_unique:
43     "(∀x. x ∈ι r ⟷ x = s) ⟷ r = {s}"
44     by(metis
45       siS_def
46       paS_is_unique)
47
48   consts seS :: "sT => (sT => bool) => sT"
49
50   syntax
51     "_seS" :: "sT => (sT => bool) => sT" ("({_./ _})")
52     translations
53     "{q. P}" == "CONST seS q P"
54
55   axiomatization where
56     Ax_seS: "∀q::sT. ∀P::(sT => bool). x ∈ι {q. P} ⟷ (x ∈ι q ∧ P x)"
57
58   function PT :: "sT => bool" where
59     "PT x = (x ∉ι {{∅}. PT})"
60     by(auto)
```

```
theorem PT_formula :
  "PT x = (x ∉ι {{∅}. PT})"
  sorry

theorem
  "{{∅}. PT} ∈ι { {{∅}. PT} }  ∧ PT {{∅}. PT} ⟶ {{∅}. PT} ∈ι {{∅}. PT}"
  by(metis
    Ax_seS
    siS_is_unique
    PT_formula)
--"Metis: The assumptions are inconsistent"



end
```

Bibliography