

```

        else collect_duplicates A_ xs (z :: ys) zs)
    else collect_duplicates A_ (z :: xs) (z :: ys) zs);
end; (*struct Example*)

```

Obviously, polymorphic equality is implemented the Haskell way using a type class. How is this achieved? HOL introduces an explicit class *equal* with a corresponding operation *equal-class.equal* such that *equal-class.equal* = (=). The preprocessing framework does the rest by propagating the *equal* constraints through all dependent code equations. For datatypes, instances of *equal* are implicitly derived when possible. For other types, you may instantiate *equal* manually like any other type class.

## 2.5 Explicit partiality

Partiality usually enters the game by partial patterns, as in the following example, again for amortised queues:

**definition** *strict-dequeue* :: 'a queue  $\Rightarrow$  'a  $\times$  'a queue **where**  
*strict-dequeue* q = (case dequeue q  
of (Some x, q')  $\Rightarrow$  (x, q'))

**lemma** *strict-dequeue-AQueue* [code]:  
*strict-dequeue* (AQueue xs (y # ys)) = (y, AQueue xs ys)  
*strict-dequeue* (AQueue xs []) =  
(case rev xs of y # ys  $\Rightarrow$  (y, AQueue [] ys))  
**by** (simp-all add: *strict-dequeue-def*) (cases xs, simp-all split: list.split)

In the corresponding code, there is no equation for the pattern *AQueue [] []*:

```

strict_dequeue :: forall a. Queue a -> (a, Queue a);
strict_dequeue (AQueue xs []) = (case reverse xs of {
    y : ys -> (y, AQueue [] ys);
});
strict_dequeue (AQueue xs (y : ys)) = (y, AQueue xs ys);

```

In some cases it is desirable to state this pseudo-“partiality” more explicitly, e.g. as follows:

**axiomatization** *empty-queue* :: 'a

**definition** *strict-dequeue'* :: 'a queue  $\Rightarrow$  'a  $\times$  'a queue **where**

$$\begin{aligned} \text{strict-dequeue}' q &= (\text{case dequeue } q \text{ of } (\text{Some } x, q') \Rightarrow (x, q') \\ &| - \Rightarrow \text{empty-queue}) \end{aligned}$$

**lemma** *strict-dequeue'-AQueue* [code]:

$$\text{strict-dequeue}' (\text{AQueue } xs \ []) = (\text{if } xs = [] \text{ then } \text{empty-queue} \\ \text{else } \text{strict-dequeue}' (\text{AQueue } [] (\text{rev } xs)))$$

$$\text{strict-dequeue}' (\text{AQueue } xs (y \# ys)) = \\ (y, \text{AQueue } xs \ ys)$$

**by** (*simp-all add: strict-dequeue'-def split: list.splits*)

Observe that on the right hand side of the definition of *strict-dequeue'*, the unspecified constant *empty-queue* occurs. An attempt to generate code for *strict-dequeue'* would make the code generator complain that *empty-queue* has no associated code equations. In most situations unimplemented constants indeed indicated a broken program; however such constants can also be thought of as function definitions which always fail, since there is never a successful pattern match on the left hand side. In order to categorise a constant into that category explicitly, use the *code* attribute with *abort*:

**declare** [[code abort: *empty-queue*]]

Then the code generator will just insert an error or exception at the appropriate position:

```
empty_queue :: forall a. a;
empty_queue = error "Foundations.empty_queue";

strict_dequeue :: forall a. Queue a -> (a, Queue a);
strict_dequeue (AQueue xs (y : ys)) = (y, AQueue xs ys);
strict_dequeue (AQueue xs []) =
  (if null xs then empty_queue
   else strict_dequeue (AQueue [] (reverse xs)));
```

This feature however is rarely needed in practice. Note that the HOL default setup already includes

**declare** [[code abort: *undefined*]]

– hence *undefined* can always be used in such situations.